HEAT EXCHANGE IN A BED OF DISPERSED MATERIAL DURING RADIAL FILTRATION

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We formulate and solve a problem in heat exchange during radial filtration of a heat-transfer agent through a bed of dispersed material. Results of a numerical solution of the problem are represented in the form of reference graphs.

In various branches of the national economy, wide use is made of cylindrical heat exchangers, in which a dispersed material located between two perforated concentric cylinders is blown past a heat-transfer agent in a radial direction (with variable velocity in a fixed bed).

The known solutions of A. Antselius and T. Shuman refer to the case of motion of a heat-transfer agent with constant velocity along the bed thickness of the material, and they are not suitable for calculating heat exchange in apparatus with radial filtration of the heat-transfer agent. In view of the fact that for a number of materials the intensity of heat exchange is directly proportional to the flow rate of the heat-transfer agent [1, 2], it is possible to solve the heat-exchange problem for radial filtration of a bed of dispersed material. We consider two variants of motion of the heat-transfer agent in the bed: from the center to the periphery and from the periphery to the center.

To simplify the solution of the problem we make the following assumptions: a) the physical properties of the material and of the heat-transfer agent are constant during heat exchange; b) the temperature gradient within individual particles is negligibly small; c) the quantity of heat transmitted between particles of the material by means of heat conduction is so small in comparision with the quantity of heat that the particles exchange with the flowing heat-transfer agent that it can be neglected; d) the coefficient of heat exchange depends linearly on the velocity of the heat-transfer agent in the bed of material.

Taking account of the symmetry of the gas flow and the assumptions that have been made, the system of differential equations of the process has the form

$$\frac{\partial \theta}{\partial \tau} = \frac{A}{r}(t-\theta), \tag{1}$$

$$\frac{\partial t}{\partial \tau} \pm \frac{c}{r}\cdot\frac{\partial t}{\partial r} = \frac{B}{r}(\theta - t)^{*},$$

where

$$A = \frac{\varkappa c}{c_{м}}, \quad B = \frac{\varkappa c}{c_{т}\varepsilon}, \quad c = v_0 r_0.$$

The problem of heat exchange for radial filtration reduces to finding in the zone $r_0 \lesseqgtr r \gtreqless r_1$; $0 < \tau < \infty$ the solution of system (1) for the conditions

$$\theta(0, \tau) = \theta_1; \; t(r_0, \tau) = t_1.$$

System (1) is reduced to canonical form by means of the following substitution of variables:

*The upper sign corresponds to motion of the air from the center to the periphery, and the lower sign corresponds to motion from the periphery to the center.

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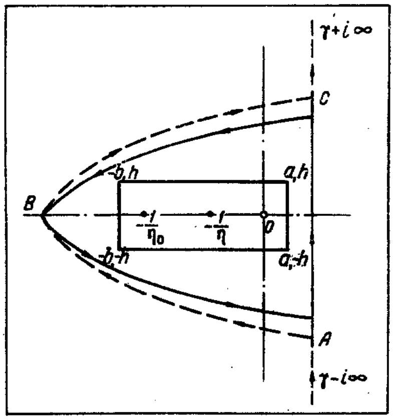

Fig. 1. Contour for calculating the integrals determining the temperature of the heat-transfer agent and the material based on Eqs. (8) and (9).

$$\xi = ab\left(c\tau \mp \frac{r^2}{2}\right) \text{ and } \eta = br,$$

where $a = A/c$; $b = B/c$; ξ and η are characteristics of system (1). In the new variables, system (1) will be

$$\eta \frac{\partial\theta}{\partial\xi} = t - \theta, \qquad (2)$$

$$\frac{\partial t}{\partial\eta} = \pm\theta \mp t.$$

In view of the fact that for the majority of practical problems $c\tau \gg r^2/2$, i.e., for $\tau = 0$ we have $\xi \approx 0$, the boundary conditions in the new variables will be

$$\theta(0, \eta) = \theta_1, \qquad (3)$$

$$t(\xi, \eta_0) = t_1, \text{ where } \eta_0 = br_0.$$

The problem with given values of the functions on the characteristics (the Goursat problem) was solved by using the method of integral Laplace transformations.

After transforming (2) and (3) according to the variable ξ and after denoting

$$\theta^* = \frac{\theta_1 - \theta}{\theta_1 - t_1}, \quad t^* = \frac{\theta_1 - t}{\theta_1 - t_1},$$

we obtain

$$\eta p\bar{\theta}^* = \bar{t}^* - \bar{\theta}; \qquad (4)$$

$$\frac{d\bar{t}^*}{p\eta} = \pm\bar{\theta}^* \mp \bar{t}^*;$$

$$\bar{\theta}^*(0, \eta) = 0$$

and

$$\bar{t}^*(p, \eta_0) = \frac{1}{p}, \qquad (5)$$

where $\bar{\theta}^*$ and $\bar{t}^*$ are Laplace transforms of the functions θ^* and t^*; p is a complex variable. The solution of system (4) for the conditions (5) gives the following dependences for the functions $\bar{t}^*$ and $\bar{\theta}^*$:

$$\bar{t}^* = \left(\frac{p\eta + 1}{p\eta_0 + 1}\right)^{\pm\frac{1}{p}} \frac{\exp(\pm\eta_0 \mp \eta)}{p}, \qquad (6)$$

$$\bar{\theta}^* = \frac{(p\eta + 1)^{\pm\frac{1}{p} - 1}}{(p\eta_0 + 1)^{\pm\frac{1}{p}}} \cdot \frac{\exp(\pm\eta_0 \mp \eta)}{p}. \qquad (7)$$

According to the transformation equation, the unknown functions t^* and θ^* are determined from the equations

$$t^*(\xi, \eta) = \frac{\exp(\pm\eta_0 \mp \eta)}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \exp\left[p\xi \pm \frac{1}{p} \ln \frac{p\eta + 1}{p\eta_0 + 1}\right] \frac{dp}{p}, \qquad (8)$$

$$\theta^*(\xi, \eta) = \frac{\exp(\pm\eta_0 \mp \eta)}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \exp\left[p\xi \pm \frac{1 \mp p}{p} \ln(p\eta + 1) \mp \frac{1}{p} \ln(p\eta_0 + 1)\right] \frac{dp}{p}. \qquad (9)$$

The integrands in (8) and (9) have the following singularities: the first-order pole $p = 0$ and the branch points $p = -1/\eta$ and $p = -1/\eta_0$. On the basis of a known assumption of the theory of functions of a complex variable, that the values of the integral $\int f(z)dz$ taken between the points z_1 and z_2 along the contours L_1 and L_2 are equal if we can convert from L_1 to L_2 owing to the continuous deformation, without intersecting a single singularity, the path of integration ($\gamma - i\infty$, $\gamma + i\infty$) in this case can be replaced by the contour ($\gamma - i\infty$, A, B, A; C, B, C, $\gamma + i\infty$) (Fig. 1). Taking into account that the integral taken

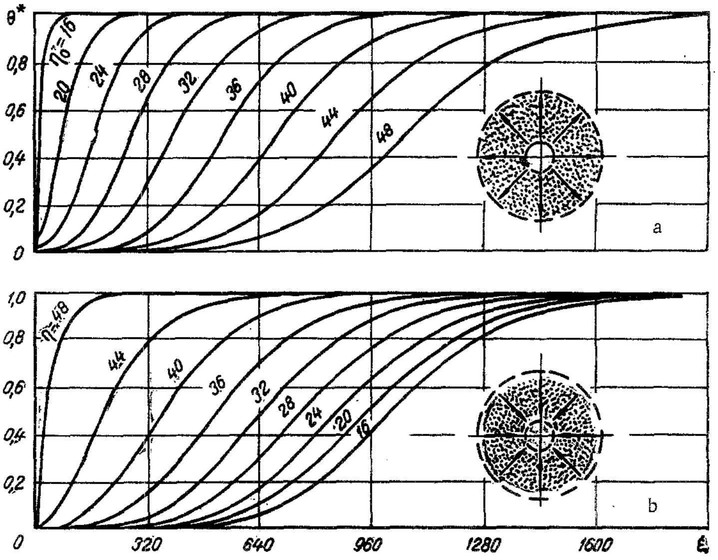

Fig. 2. Graphs for calculating the temperature of the material for motion of a heat-transfer agent from the center to the periphery (a) and from the periphery to the center (b).

along the contour ($\gamma - i\infty$, A, B, C, $\gamma + i\infty$) equals zero [3], to obtain real integrals we can use the rectangular contour ($a + ih$, $-b + ih$, $-b - ih$, $a - ih$), which is obtained as a result of the deformation of the contour ABC.

As a result of calculating the integrals along this contour, according to (8) and (9), we obtain the following expressions for the temperature of the material and of the heat-transfer agent for radial filtration: temperature of the heat-transfer agent

$$t^*(\xi,\eta) = -\frac{\exp(\pm\eta_0\mp\eta)}{\pi}\int_0^b\left(\frac{m_1}{n_1}\right)^{\mp\frac{x}{x^2+h^2}}\exp\left(-x\xi\pm\frac{h}{x^2+h^2}A_1\right)\times$$
$$\times\frac{\sin\alpha\,dx}{\sqrt{x^2+h^2}} - \frac{\exp(\pm\eta_0\mp\eta-b\xi)}{\pi}\int_0^h\left(\frac{m_2}{n_2}\right)^{\mp\frac{b}{b^2+y^2}}\times$$
$$\times\frac{\exp\left(\pm\frac{y}{b^2+y^2}A_2\right)\cos\beta\,dy}{\sqrt{b^2+y^2}} - \frac{\exp(\pm\eta_0\mp\eta)}{\pi}\int_0^a\left(\frac{m_3}{n_3}\right)^{\pm\frac{x}{x^2+h^2}}\times$$
$$\times\frac{\exp\left(x\xi\pm\frac{h}{x^2+h^2}A_3\right)\sin\gamma\,dx}{\sqrt{x^2+h^2}} + \frac{\exp(\pm\eta_0\mp\eta+a\xi)}{\pi}\times$$
$$\times\int_0^h\left(\frac{m_4}{n_4}\right)^{\pm\frac{a}{a^2+y^2}}\frac{\exp\left(\pm\frac{y}{a^2+y^2}A_4\right)\cos\omega\,dy}{\sqrt{a^2+y^2}}; \qquad (10)$$

temperature of the material

$$\theta^*(\xi,\eta) = -\frac{\exp(\pm\eta_0\mp\eta)}{\pi}\int_0^b\frac{1}{m_1}\left(\frac{m_1}{n_1}\right)^{\mp\frac{x}{x^2+h^2}}\times$$
$$\times\frac{\exp\left(-x\xi\pm\frac{h}{x^2+h^2}A_1\right)\sin(\alpha-\alpha^*)\,dx}{\sqrt{x^2+h^2}} - \frac{\exp(\pm\eta_0\mp\eta-b\xi)}{\pi}\times$$
$$\times\int_0^h\frac{1}{m_2}\left(\frac{m_2}{n_2}\right)^{\mp\frac{b}{b^2+y^2}}\frac{\exp\left(\pm\frac{y}{b^2+y^2}A_2\right)\cos(\beta-\beta^*)\,dy}{\sqrt{b^2+y^2}} -$$

$$-\frac{\exp(\pm\eta_0\mp\eta)}{\pi}\int_0^a \frac{1}{m_3}\left(\frac{m_3}{n_3}\right)^{\pm\frac{x}{x^2+h^2}}\times$$

$$\times\frac{\exp\left(x\xi\pm\frac{h}{x^2+h^2}A_3\right)\sin(\gamma-\gamma^*)\,dx}{\sqrt{x^2+h^2}}+\frac{\exp(\pm\eta_0\mp\eta+a\xi)}{\pi}\times$$

$$\times\int_0^h \frac{1}{m_4}\left(\frac{m_4}{n_4}\right)^{\pm\frac{a}{a^2+y^2}}\frac{\exp\left(\pm\frac{y}{a^2+y^2}A_4\right)\cos(\omega-\omega^*)\,dy}{\sqrt{a^2+y^2}}, \quad (11)$$

where

$$m_1=\sqrt{(1-x\eta)^2+(\eta h)^2};\quad m_2=\sqrt{(1-b\eta)^2+(\eta y)^2};$$
$$m_3=\sqrt{(1+x\eta)^2+(\eta h)^2};\quad m_4=\sqrt{(1+a\eta)^2+(\eta y)^2};$$
$$n_1=\sqrt{(1-x\eta_0)^2+(\eta_0 h)^2};\quad n_2=\sqrt{(1-b\eta_0)^2+(\eta_0 y)^2};$$
$$n_3=\sqrt{(1+x\eta_0)^2+(\eta_0 h)^2};\quad n_4=\sqrt{(1+a\eta_0)^2+(\eta_0 y)^2};$$
$$A_1=\operatorname{arctg}\frac{\eta h}{1-x\eta}-\operatorname{arctg}\frac{\eta_0 h}{1-x\eta_0};$$
$$A_2=\operatorname{arctg}\frac{\eta y}{1-b\eta}-\operatorname{arctg}\frac{\eta_0 y}{1-b\eta_0};$$
$$A_3=\operatorname{arctg}\frac{\eta h}{1+x\eta}-\operatorname{arctg}\frac{\eta_0 h}{1+x\eta_0};$$
$$A_4=\operatorname{arctg}\frac{\eta y}{1+a\eta}-\operatorname{arctg}\frac{\eta_0 y}{1+a\eta_0};$$
$$\alpha=\xi h\mp\frac{h}{x^2+h^2}\ln\frac{m_1}{n_1}\mp\frac{x}{x^2+h^2}A_1+\operatorname{arctg}\frac{h}{x};$$
$$\alpha^*=\operatorname{arctg}\frac{\eta h}{1-x\eta};$$
$$\beta=\xi y\mp\frac{y}{b^2+y^2}\ln\frac{m_2}{n_2}\mp\frac{b}{b^2+y^2}A_2+\operatorname{arctg}\frac{y}{b};$$
$$\beta^*=\operatorname{arctg}\frac{\eta y}{1-b\eta};$$
$$\gamma=\xi h\mp\frac{h}{x^2+h^2}\ln\frac{m_3}{n_3}\pm\frac{x}{x^2+h^2}A_3-\operatorname{arctg}\frac{h}{x};$$
$$\gamma^*=\operatorname{arctg}\frac{\eta h}{1+x\eta};$$
$$\omega=\xi y\mp\frac{y}{a^2+y^2}\ln\frac{m_4}{n_4}\pm\frac{a}{a^2+y^2}A_4-\operatorname{arctg}\frac{y}{a};$$
$$\omega^*=\operatorname{arctg}\frac{\eta y}{1+a\eta}.$$

Based on Eq. (11), using a program for calculating integrals of rapidly oscillating functions on an M-200 computer, we calculate the quantities θ^* for two variants of motion of air in the bed (Fig. 2). The limits of variation of the parameters are η = 16-48; ξ = 0-1800.

Analysis of the calculated data shows that radial filtration of a heat-transfer agent in a bed of dispersed material is characterized by a variable heat-exchange zone. For motion of the heat-transfer agent from the center to the periphery, the heat-exchange zone decreases in time, and for the opposite motion it increases. The graphs presented can be used for designing radial heat exchangers with a fixed bed of dispersed material and for estimating the efficiency of operating apparatuses of similar construction.

NOTATION

θ, t are the temperatures of the material and of the heat-transfer agent;
τ is the time;

t_1, θ_1	are the temperature of the heat-transfer agent at the bed inlet and the initial temperature of the material;
r, r_0, r_1	are the radial coordinate along the bed thickness and the radii of cylinders corresponding to the bed inlet and the heat-transfer-agent outlet from the bed of material;
v_0	is the inlet velocity of the heat-transfer agent;
C_M, C_T	are the volumetric specific heat of the bed material and of the heat-transfer agent;
$\varkappa$	is the coefficient of proportionality between the volumetric heat-transfer coefficient and the velocity of the heat-transfer agent;
ε	is the coefficient of porosity.

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